



Population Riddles

Millions & Billions

The **significance of large numbers** can be hard to understand. The global population is made up of 8 billion people, and the population of the United States is over 330 million. This exercise will help you appreciate the difference between millions and billions:

Your rich uncle has just left you one billion dollars. If you accept the money you must count it for eight hours a day at the rate of one dollar per second. When you are finished counting, the billion dollars will be yours and only then may you begin to spend it.

Do you accept your uncle's offer?

Why or why not?

How long would it take to count a billion dollars at this rate?

How long would it take to count a million dollars at the same rate?

How old are you if you are a million seconds old?

How old are you if you are a billion seconds old?

Bacteria Bottles

Doubling time is the time it takes a population to double at a constant rate of growth. Bacteria, for instance, multiply by division. One bacterium becomes two. Then two divide into four; the four divide into eight, and so on. For a certain strain of bacteria, the time for this division process is one minute.

If you put one of these bacterium in a bottle at 11:00 p.m., the entire bottle will be full by midnight. When would the bottle be half full?

How do you know?

Suppose you could be a bacterium in this bottle. At what time would you first realize that you were running out of space? Suppose that at 11:58 some bacteria realize that they are running out of space in the bottle. So they launch a search for new bottles. They look far and wide, and finally, offshore in the Arctic Ocean, they find three new empty bottles. Great sighs of relief come from all the bacteria. This is three times the number of bottles they've known. Surely, they think, their space problems are over. Is that so?

Since their space resources have quadrupled, how long can their growth continue?

Calendar Riddle

Exponential growth is a constant rate of growth applied to an increasing base. Doubling a small number over and over soon means doubling ever-larger numbers.

A father complained that his son's allowance of \$5 per week was too much. The son replied, "Okay, Dad. How about this? You give me a penny for the first day of the month, 2 cents for the second, 4 cents for the next, 8 cents for the next, and so on for every day of the month." The father readily consented.

Who was the more clever?

What would the son's allowance be on day 31?

Bacteria Bottles was taken from 101 Ways to Teach About Exponential Growth and Its Consequences, available free from the Tri-County Teacher Education Center, Sebring, FL 33870



Population Riddles Answers

Millions & Billions

The significance of large numbers can be hard to understand. The global population is made up of 8 billion people, and the population of the United States is over 330 million. This exercise will help you appreciate the difference between millions and billions:

Your rich uncle has just left you one billion dollars. If you accept the money you must count it for eight hours a day at the rate of one dollar per second. When you are finished counting, the billion dollars will be yours and only then may you begin to spend it.

Do you accept your uncle's offer? *No.*

Why or why not? *Because it would take too long to count the money.*

How long would it take to count a billion dollars at this rate? *Over 95 years.*

How long would it take to count a million dollars at the same rate? *About 35 days.*

How old are you if you are a million seconds old? *About 11 1/2 days old.*

How old are you if you are a billion seconds old? *About 31 1/2 years old.*

Bacteria Bottles

Doubling time is the time it takes a population to double at a constant rate of growth. Bacteria, for instance, multiply by division. One bacterium becomes two. Then two divide into four; the four divide into eight, and so on. For a certain strain of bacteria, the time for this division process is one minute.

If you put one of these bacterium in a bottle at 11:00 p.m., the entire bottle will be full by midnight. When would the bottle be half full? *At 11:59.*

How do you know? *Because the bottle is full at midnight and the doubling time is one minute.*

Suppose you could be a bacterium in this bottle. At what time would you first realize that you were running out of space? *Answers will vary. To clarify, ask students: "At 11:55, when the bottle was only 3 percent full and 97 percent empty, would it be easy to perceive that there was a space problem?"*

Suppose that at 11:58 some bacteria realize that they are running out of space in the bottle. So they launch a search for new bottles. They look far and wide, and finally, offshore in the Arctic Ocean, they find three new empty bottles. Great sighs of relief come from all the bacteria. This is three times the number of bottles they've known. Surely, they think, their space problems are over. Is that so? *No.*

Since their space resources have quadrupled, how long can their growth continue? The bacteria's growth can continue at its current rate until 12:02 p.m.:

At 11:58, the first bottle is one-quarter full. By 11:59, the bacteria will have doubled to fill the first bottle half-way. By midnight, bottle 1 will be completely full. By 12:01, the bacteria in bottle 1 will have doubled and will fill up bottle 2. By 12:02, the bacteria in bottles 1 and 2 will each have doubled, to fill bottles 3 and 4.

*11:54 p.m. Bottle 1 is 1.5% full.
 11:55 p.m. Bottle 1 is 3% full.
 11:56 p.m. Bottle 1 is 6.25% full.
 11:57 p.m. Bottle 1 is 12.5% full.
 11:58 p.m. Bottle 1 is 25% full.
 11:59 p.m. Bottle 1 is 50% full.
 Midnight Bottle 1 is 100% full.
 12:01 a.m. Bottles 1 and 2 are full.
 12:02 a.m. Bottles 1, 2, 3 and 4 are all full.*

Calendar Riddle

Exponential growth is a constant rate of growth applied to an increasing base. Doubling a small number over and over soon means doubling ever-larger numbers.

A father complained that his son's allowance of \$5 per week was too much. The son replied, "Okay, Dad. How about this? You give me a penny for the first day of the month, 2 cents for the second, 4 cents for the next, 8 cents for the next, and so on for every day of the month." The father readily consented.

Who was the more clever? *The son was the more clever.*

What would the son's allowance be on day 31? *On day 31 alone, his allowance would be \$10,737,418.24. His total allowance for the month would be \$21,474,836.47.*

1 \$.01	2 \$.02	3 \$.04	4 \$.08	5 \$.16	6 \$.32	7 \$.64
8 \$1.28	9 \$2.56	10 \$5.12	11 \$10.24	12 \$20.48	13 \$40.96	14 \$81.92
15 \$163.84	16 \$327.68	17 \$655.36	18 \$1,310.72	19 \$2,621.44	20 \$5,242.88	21 \$10,485.76
22 \$20,971.52	23 \$41,943.04	24 \$83,886.08	25 \$167,772.16	26 \$335,544.32	27 \$671,088.64	28 \$1,342,177.28
29 \$2,684,354.56	30 \$5,368,709.12	31 \$10,737,418.24				