DOUBLE FEATURE

People and the Planet
Lessons for a Sustainable Future

INTRODUCTION

Doubling time (and it's opposite, half-life) are important mathematical and science concepts that help us understand trends in biology, ecology and economics. Bacteria populations that divide rapidly, or money invested at a fixed interest rate, are quantities that can grow **exponentially**. This means that the larger they get, the faster they grow. With a short "doubling time," or amount of time it takes the quantity to double, even a small quantity can rapidly become enormous. Doubling time is also a concept used in **demography** (the study of population). Knowing the rate at which a population is growing or shrinking enables demographers to project the years it will take that population to double or reduce by half.

MATERIALS

- Student Worksheets 1 & 2
- Calculator

PART 1: WHICH PAYCHECK?

PROCEDURE

1. Pose the following question to your students. Ask them to first give you a quick answer on which option they would choose. Then provide them time to work through the math and ask them again.

   "You are finally offered your very first job! But the pay is up to you. You have the choice between receiving $100 on the first day, $200 on the second day and so on (each day is $100 more than the last) OR you can receive one cent on the first day, two cents the second day, four cents the third day, and so on, doubling your pay each day through the end of a 30-day month. Which do you choose?"

   Answer: The first choice adds up to $46,500; the second choice adds up to $10,737,418.23

CONCEPT:
A small amount, when continually doubled, can grow into a very large amount quickly. Understanding "doubling time" can also help us understand the impacts of population growth rates over time.

OBJECTIVES:
Students will be able to:

- Calculate the doubling of an amount and understand how small amounts build quickly with multiple doublings.
- Explain how birth and death rates affect population change.
- Explain the difference between arithmetic (linear) and geometric (or exponential) growth.
- Calculate the rate of natural increase for several countries, as well as their corresponding population doubling or halving times.
- Analyze countries' projected population doubling times to project future challenges of demographic changes.

SUBJECTS:
Math, science (life), social studies (geography, economics)

SKILLS:
Calculating percentages, calculating doubling time and half-life, using algebraic formulas and exponents, using scientific notation

METHOD:
Students use mathematical formulas to calculate growth rates and doubling times, and to determine the difference between arithmetic and geometric growth.

2. Ask them how they would describe the kind of numerical progression each option presents. Explain that the first option is an example of **arithmetic (or linear) growth**, where you add (or subtract) a set amount (in this case +$100) each day. The second option is an example of **geometric growth**, where you multiply (or divide) by a set amount (in this case x2) each day.

PART 2: THE KING'S REWARD

PROCEDURE

1. Distribute copies of Student Worksheet 1 to students. This could be completed in class or for homework. Go over the Worksheet as a class.

    Note: You can also find this story in book form. *The King's Chessboard* by David Birch is a picture book recommended for grades 2-6 but can provide useful illustrations of the math concepts for middle schoolers.

Answers to Student Worksheet 1

1. *Answers will vary. Perhaps the mathematician thought that food was the most valuable resource in the kingdom.*
2. *Answers will vary. Students may think the king might become concerned when the number of rice grains per square reaches the millions.*
3. *Students may suggest doubling the amount of each previous square until you get to the end.*
4. *Students may come up with a formula like the one described at the bottom of the Worksheet.*
5. 9.2×10^{18}

PART 3: ON THE DOUBLE

Now turn students' newly formed doubling skills to population. Like the grains of rice on the chessboard, world population started small – there were only about 170 million people 2,000 years ago (roughly half the population in the U.S. today). The population grew slowly until about 200 years ago when we had 1 billion people. Then population started to increase more quickly. At its peak in 1963, the world population growth rate was 2.2 percent.[1] That seems like a pretty small number. But, because there were 3.2 billion people on the planet at that time, a growth rate of 2.2 percent meant we added over 70 million to our global population that year alone.

Go over the math used to calculate the number of people added to the human population in 1963.

3.2 billion = 3,200,000,000 and 2.2% = 0.022
3,200,000,000 x 0.022 = 70,400,000 people

By 2020, the population growth rate dropped to 1.1 percent. However, because the planet was then home to 7.8 billion people, the slow growth rate still added more than 85 million people that year.

7,800,000,000 x 0.011 = 85,800,000 people

Note: For a visual of this growth, students can watch the 6 minute video, *World Population*, at www.WorldPopulationHistory.org.

PROCEDURE

1. Explain to students that the larger the difference between a country's **birth rate** and its **death rate**, the greater the population change. (A larger birth rate creates a positive growth rate; a larger death rate creates a negative growth rate.) Distribute Student Worksheet 2. Have students look at each country's birth and death rate to determine which one will have the highest rate of increase. Which countries' population will decline at their current rates? How do you know?
Answer: Nigeria and Guatemala have the highest rates because the difference between the birth and death rates are greatest. Germany, Japan and Bulgaria will have a decline in population because their death rates are higher than their birth rates.

2. Now have students calculate the rate of annual natural increase for each country's population on the Worksheet, using the following formula:

(Birth Rate – Death Rate)/10 = Annual Natural Increase (%)

Explain that birth and death rates are expressed per 1,000, but percentages are based on 100. So, after subtracting the death rate from the birth rate, you divide by 10.

Provide the world's annual population increase as an example. In 2020 the world's birth rate was 18.5 births per 1,000 people and the death rate was 7.5 deaths per 1,000 people.[2] So, the world's annual population increase in 2020 can be calculated with the following equation:

$$(18.5 - 7.5)/10 = 11/10 = 1.1\ \%$$

3. Based on the **rate of natural increase**, students can now calculate each country's population doubling time or halving time with the following formula. (Keep in mind that these figures will reflect growth due to natural increase alone, without considering immigration or emmigration.)

70/Rate of Increase = Doubling Time or Halving time (in years)

Note: 70 is 100 times the natural logarithm of 2 which is used when solving the equation $ar^x = 2a$ where "a" is the initial population and "r" is the growth factor.

For example, calculate the doubling time for the world population at the present rate of increase with the following formula:

$$70/1.1 = 64 \text{ years}$$

Answers to Student Worksheet 2

Country	Birth Rate (2020)	Death Rate (2020)	Annual Natural Increase (%)	Doubling or Halving Time (in years)
Bulgaria	9	16	-0.7	100 (halving)
China	10	7	0.3	233
Germany	10	12	-0.2	350 (halving)
Guatemala	22	5	1.7	41
India	20	6	1.4	50
Japan	7	11	-0.4	175 (halving)
Nigeria	37	12	2.5	28
South Africa	20	9	1.1	63
South Korea	6	6	0	N/A
United States	12	9	0.3	233

Source: Population Reference Bureau

DISCUSSION QUESTIONS

1. Which figures differ most among countries, the birth rates or the death rates? How would you explain this?

   Birth rates vary more because there is more variability in family size. The birth rate is higher when the average family size is larger. For example, Nigeria (with a birth rate of 37/1,000) averages 5.3 children per family while Germany (with a birth rate of 10/1,000) averages 1.6 children per family.. Death rates are affected by age distribution and most countries will eventually show a rise in the overall death rate, in spite of continued decline in mortality at all ages, as declining fertility results in an aging population.

2. What do you think accounts for variations in death rates among countries? Why do you think the death rate in the U.S. is higher than in Guatemala?

   A variety of factors can influence death rates including wars, disease, famines and poor health care. The difference in death rates between Guatemala and the U.S. can be attributed to the U.S. having a larger proportion of elderly citizens.

3. What would likely be some challenges for government leaders in Guatemala and Nigeria in planning for a doubled population in 20-40 years?

   In planning for twice as many country residents, those leaders would need to consider strategies for increasing all sorts of infrastructure – food systems, housing, education, transportation, energy, healthcare, employment and more.

4. Are countries like Germany or Bulgaria likely to run out of people? Why or why not?

   Not likely. Birth and death rates can change each year due to various social and economic factors. There *are also factors outside of the natural increase. For example, the recent influx of hundreds of thousands of migrants to Germany from war-torn countries in the Middle East is likely to change Germany's demographic profile.*

5. Would country leaders have concerns if their country is losing population? Why or why not?

   Perhaps. They may fear labor shortages or a growing proportion of elderly who require more healthcare and home services. On the other hand, a decreasing population could alleviate crowding, unemployment or strains on land and water resources.

6. Do you think that the low growth rate countries in Europe and East Asia balance out the higher growth rate countries in other parts of the world?

   Of the nearly 200 countries, only 29 are at zero or negative population growth. These 29 countries represented 732 million people in 2020, less than 7 percent of the world's population. Because most countries are experiencing population growth (with the fastest growth in sub-Saharan Africa), the United Nations projects world population will continue growing through this century to between 10 and 11 billion by 2100.

ASSESSMENT

Students solve the following money riddle and explain their answers in writing. Then solve the word problems to show their understanding of population doubling times.

1. Which of the following situations would generate the most money after 20 years? Which would you choose? Justify your answer.

   a. Receive $200 a year for the first year, $400 a year for the second year, $800 a year the third year, and so forth, doubling each year.
   b. Receive $20,000 a year for 20 years.

2. You are the mayor of a city of 100,000 people. If your city's population is growing by 2 percent each year, what is that population's doubling time? What challenges might arise in the community in the face of this population change?

3. Would the doubling time change if you started with a population of just 10,000? Why or why not?

Answers

1. *Option "a" would produce the most money after 20 years: $209,715,000 as compared with $400,000 for choice "b."*
2. *At a 2 percent growth rate, the city of 100,000 would double to 200,000 in 35 years. This is not a lot of time to provide the infrastructure and services (jobs, schools, medical care, transportation, energy, etc.) for a doubled population.*
3. *No. Regardless of the initial population size, the doubling time is the same.*

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[1] United Nations. (2015). *World Population Prospects: 2015 Revision*. Retrieved from https://population.un.org/wpp/Publications/Files/Key_Findings_WPP_2015.pdf

[2] Population Reference Bureau. (2020). *2020 World Population Data Sheet*. [Wall chart]. Retrieved from https://interactives.prb.org/2020-wpds

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STUDENT WORKSHEET 1

Name: ______________________________ Date: ______________

There's an ancient story about the invention of the game of chess. The story goes that a king in India many centuries ago grew weary of his games and asked a poor mathematician to come up with a new game for him to play. The mathematician invented chess and the king was so enamored with this new game that he asked the mathematician to name his reward. The mathematician answered humbly for the king to place a grain of rice on the first square of the chessboard and then two grains on the next and continue to double the number of grains on every subsequent square of the chessboard until all 64 squares contained rice.

			8	16	32	64	128
256	512	1024	2048	4096	8192	16,384	32,768
65K	131K	262K	524K	1M	2M	4M	8M
16M	33M	67M	134M	268M	536M	1B	2B
4B	8B	17B	34B	68B	137B	274B	549B

(K = Thousand; M = Million; B = Billion)

1. The king was astounded that the mathematician didn't ask for gold or jewels. Why do you think the mathematician chose the reward he did?

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2. Is there a point when the king might regret his decision to honor the mathematician's request? If so, how far along the chessboard (at which square) might the king become concerned that he cannot fulfill the reward?

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3. How would you determine the number of grains of rice the mathematician would receive in the 64th square?

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4. Is there a quicker way than doubling the quantity 63 times by hand or on your calculator? Explain.

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Exponents to the Rescue!

The doubling scenario above - starting with a small amount and doubling it repeatedly - is an example of geometric or exponential growth. The small amount soon becomes very large. The doubling can be expressed in this formula:

$y = 2^{(x-1)}$ (where x = the square number). So, finding the quantity of rice grains on the 64th square would involve 63 doublings starting with the quantity on the first square: $y = 2^{(64-1)}$ or $y = 2^{63}$

This can be accomplished using the exponent function on your calculator (x^y key on your computer or on a calculator app on your phone or tablet; ^ key on a TI-84, as in 2 ^ 63 to calculate).

5. How many grains of rice would the mathematician receive on the 64th square? (Note, it's best to express this using scientific notation.)

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STUDENT WORKSHEET 2

Name: ____ Date: ____________________

Use the formulas below to calculate the annual rate of natural increase and the doubling or halving times for the populations of the countries listed on the chart below.

% Annual Increase = (Birth Rate – Death Rate)/10 Doubling Time (in years) = 70/% Annual Increase

Country	Birth Rate (2020)	Death Rate (2020)	Annual Natural Increase (%)	Doubling or Halving Time (in years)
Bulgaria	9	16		
China	10	7		
Germany	10	12		
Guatemala	22	5		
India	20	6		
Japan	7	11		
Nigeria	37	12		
South Africa	20	9		
South Korea	6	6		
United States	12	9		