MEASURING A MILLION

People and the Planet
Lessons for a Sustainable Future

INTRODUCTION

There are close to 8 billion people living on Earth today and by 2050, we are expected to reach a world population of over 9 billion. Students and adults alike have difficulty conceptualizing such large numbers. Using familiar context, such as distance, time, or height can help students visualize just how big these large numbers are and see the difference in magnitude between a million and a billion.

MATERIALS

- Student Worksheets 1 – 5
- Meter sticks
- Stack of 50 sheets of paper
- Classroom desk
- Map of the United States
- Calculators (optional)

PART 1: BIG RIDDLES PROCEDURE

1. Share the following riddles with your students. You can have students guess or give estimates, or for more of a challenge, have students work out the answers in small groups or together as a class.

   a. Your rich uncle has just died and has left you $1 billion, with a catch: In order to accept the money, you must count it for eight hours a day at the rate of $1 per second. When you are finished counting, the $1 billion is yours and then you may start to spend it.

   Do you accept your uncle's offer? Why or why not? How long will it take to count the money?
   Answer: Over 95 years. 1,000,000,000/60 seconds = 16,666,667 minutes/60 minutes = 277,778 hours/8 hours = 34,722 days/365 days = 95.13 years

CONCEPT

To understand the implications of population size, it is helpful for students to be able to conceptualize large numbers like millions and billions.

OBJECTIVES

Students will be able to:

- Solve one of four math problems in a cooperative learning group which requires them to measure distance, area or volume of millions (and billions) of items or people.
- Solve riddles illustrating the difference between million and a billion.

SUBJECTS

Math

SKILLS

Calculating with large numbers, estimating, measuring lengths/areas/volumes, averaging, using simple algebraic formulas, converting units

METHOD

Through riddles and cooperative learning math activities, students work through problems to calculate and visualize millions and billions of things and people.

b. What if your uncle offered you $1 million, but you need to count the money at the same rate (one dollar per second, eight hours per day)? Would you accept your uncle's offer now? How long would it take to count?
Answer: About 35 days. 1,000,000/60 seconds = 16,667 minutes/60 minutes = 278 hours/8 hours = 34.7 days

c. If you spent $1 million at the rate of $1,000 per day, how long would it take you to spend it?
Answer: 1,000 days, or about 2 $^{3}/_{4}$ years. *1,000,000/1,000 = 1,000 days/365 days = 2.74 years*

d. How long would it take you to spend $1 billion at the same rate?
Answer: 1 million days, or 2,739.7 years. 2.74 years x 1,000 = 2,740 years

e. How old would you be if you were a million seconds old?
Answer: About 12 days old. 1,000,000/60 seconds = 16,667 minutes/60 minutes = 278 hours/24 hours = 11.6 days

f. How old would you be if you were a billion seconds old?
Answer: About 32 years old. 1,000,000,000/60 seconds = 16,666,667 minutes/60 minutes = 277,778 hours/24 hours = 11,574 days/365 days = 31.7 years OR 11.6 days x 1,000 = 11,600 days/365 days = 31.8 years.

PART 2: MEASURING A MILLION

PROCEDURE

1. Divide the class into groups of 4-5 students. Each group will be responsible for doing one of the following five activities, using the materials listed on page 1.

2. Distribute the Student Worksheets if you want your students to have the benefit of the suggested steps listed for each problem. If you would prefer to have students devise their own steps, then you may choose to simply write the groups' challenges on the board. When the activities are finished, have each group present their methods and findings to the class.

Group #1: Using the stack of 100 sheets of paper, estimate how tall a stack of a million sheets of paper would be. How tall would a stack of a billion sheets of paper be? If we tipped the stack of a million sheets of paper over, where would it stretch to? If we tipped the stack of a billion sheets of paper over, where would it stretch to? (You choose the direction.)

Group #2: What would be the length of a million people holding hands? How about the length of a billion people holding hands? Where would a chain of a million people holding hands extend to starting from this classroom? Where would a chain of a billion people holding hands extend to? (You choose the direction.)

Group #3: If you took a million steps starting from the door of the classroom, how many miles would you travel? Choose a direction and determine where you would end up. How many miles would you go if you took a billion steps in the same direction? Where would you be?

Group #4: Imagine a crowd of a million people. How big a field do you think you would need to contain all of them? How much land area would you need to hold a billion people? If we looked at Lambeau Field in Green Bay, WI, a football stadium that can seat 81,435 people, how many football stadiums would we need to seat a million people? How many stadiums would seat a billion people?

Group #5: How many desks would fit in the classroom? How many classrooms would it take to hold a million desks? How many classrooms would we need to hold a billion desks?

Note: The Group 5 activity involves calculating volume and is more challenging and time-consuming than the other four.

Answers to Student Worksheets

Group #1: *Answers may vary depending on the thickness of the paper. We'll assume 100 sheets of paper is 10 mm, or 1 cm, thick (roughly the thickness of 20# copy paper). So a million sheets of paper is 10,000 cm or 100 m tall, roughly the height of the Statue of Liberty, from the ground to the tip of the torch. It's also about the length of a football field, which is 109 m. If we tipped it over, it would reach 0.1 km or 0.06 miles.*

A billion sheets of paper is 1,000 times the height of a million sheets (100 m) so it's 100,000 m tall. (A billion is 1,000 million.) A height of 100,000 m (or 100 km) is from sea level on Earth to the edge of outer space. If we tipped that stack over, it would be the length of 1,000 football fields. 100,000 m (100 km) from Washington DC is 62 miles. As the crow flies, from Washington DC to Hagerstown, Maryland or Gettysburg, Pennsylvania.

Group #2: *Answers may vary depending on how far apart people stand when holding hands. Assuming the length of four people holding hands is 3 m, students will cross multiply:*

$$\frac{4\text{ people}}{3\text{ m}} = \frac{1{,}000{,}000}{x\text{ meters}} = 3{,}000{,}000/4 = 750{,}000\text{ m} = 750\text{ kilometers (466 miles)}$$

The distance of a million people holding hands would stretch from New York City to Virginia Beach, VA. A chain of a billion people holding hands is 1,000 times 750 km or 750,000 km. The chain of people holding hands would extend 18 times around the Earth (the circumference of the Earth is 40,705 km or 25,293 miles) and is greater than the diameter of the sun.

Group #3: *Answers may vary depending on the length of a person's steps. Assuming that a person's step is 70 cm, or 2.3 feet, 2.3 feet/person x 1,000,000 people = 435.6 miles [2,300,000 ft/5,280 ft]. This is approximately the driving distance from New York City to Raleigh, NC or New York City to Quebec, Canada.*

One billion steps would be 1,000 x 4,35.6 miles = 435,600 miles which is nearly the distance to the moon and back (477,710 miles). The Earth's circumference is about 25,000 miles. 435,600 miles/25,000 miles = 17.4 times around the Earth.

Group #4: *Assuming a person needs 2 sq. ft. of space or 0.19 m^2. 0.19 m^2 x 1,000,000 = 190,000 m^2 (or 46.9 acres, about the size of a small college campus). A billion people would need 0.19 m^2 x 1,000,000,000 = 190,000,000 m^2 of space or about the size of the country of Syria or Washington State (46,900 acres).*

13 stadiums would seat a million people [1,000,000/81,435= 12.27] and 12,280 stadiums would seat a billion people [1,000,000,000/81,435= 12,279.73]

Group #5: *A standard desk is 61 cm x 45 cm x 76 cm or 208,620 cm^3. Because there are 1,000,000 cm^3 in one cubic meter, a standard desk is 0.208620 m^3. The volume of a standard classroom in 300 m^3. Divide the room volume by the desk volume: 300 m^3/0.208620 m^3 = approximately 1,438 desks could fit into one classroom.*

It would take 695 classrooms to fit a million desks [1,000,000/1,438 = 695] and 695,410 classrooms to fit a billion desks [1,000,000,000/1,438 = 695,410]. You would need 6,954 schools of 100 classrooms each.

ASSESSMENT

Monitor each group's discussion to ensure all students are actively engaged and evaluate their brief presentations from Part 2 to ensure comprehension.

FOLLOW-UP ACTIVITIES

1. There are nearly 8 billion people in the world. What would be the length of a row of 8 billion people standing side by side?

2. A sense of scale helps us better understand large numbers. In 2020, tennis star Roger Federer was the highest paid athlete in the world, earning $106 million in salary, bonuses, and endorsements over a 12-month period. (Forbes) How would you scale down his annual salary of $106 million dollars to a smaller number so that it's easier to understand – maybe in terms of the amount of dollars per hour or day?

MEASURING A MILLION

STUDENT WORKSHEET 1

Name: ____ Date: ____________________

Group #1: Using the stack of 100 sheets of paper, estimate how tall a stack of a million sheets of paper would be. How tall would a stack of a billion sheets of paper be? If we tipped the stack of a million sheets of paper over, where would it stretch to? If we tipped the stack of a billion sheets of paper over, where would it stretch to? (You choose the direction.)

Hint: 1 km = 0.62 miles

1. I would estimate a stack of a million sheets of paper would be ________________ cm. The average estimate for my group is ________________ cm.

2. The actual height of a stack of 100 sheets of paper is ________________ mm or ________________ cm.

3. Based on the height of #2, a million sheets of paper would be ________________ mm or ________________ cm tall. Convert this to meters: ________________ m

$$\frac{\text{100 sheets}}{\text{\_\_\_ cm (answer from \#2)}} = \frac{\text{1 million sheets}}{\text{x cm}}$$

4. Based on the height of #3, 1,000,000,000 sheets of paper would be 1,000 times the height of 1,000,000 sheets of paper. So, a billion sheets of paper would be ________________ m tall.

5. If we laid over the stack of a million sheets of paper, it would stretch ________________ km, or ________________ miles, which would roughly be the distance from ____________________ to ______________________________.

6. If we laid over the stack of a billion sheets of paper, it would stretch ________________ km, or ________________ miles. (1,000 times the answer to #5). This would be the distance from ______________________________ to ______________________________.

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STUDENT WORKSHEET 2

Name: ______________________________ Date: ________________

Group #2: What would be the length of a million people holding hands? How about the length of a billion people holding hands? Where would a chain of a million people holding hands extend to starting from this classroom? Where would a chain of a billion people holding hands extend to? (You choose the direction.)

Hint: 1 km = 0.62 miles

1. The length of 4 people holding hands is ____________ meters.

The ratio of people to length is ____________ : ____________. Cross multiply

to solve for x.

$$\frac{4 \text{ people}}{\_\_\_ \text{ meters}} = \frac{1{,}000{,}000 \text{ people}}{x \text{ meters}}$$

2. Based on the answer from #1, the length of 1,000,000 people holding hands is ____________.

The length of a billion people holding hands is 1,000 times a million people holding

hands: ____________

3. Looking at a map of your state or country, can you find the distance between two cities which is equivalent to the length of a million people holding hands? ____________________ to ____________________

4. Look at a map of your area. Where would a chain of a million people holding hands extend

north? ________________ A million steps south? ________________

East? ____________________ West? ____________________

5. A chain of a billion people holding hands would be ____________________ miles long.

6. The circumference of the Earth is roughly 25,000 miles. A line of a billion people holding hands would

circle the globe how many times? ____________________

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STUDENT WORKSHEET 3

Name: ____ Date: ____________________

Group #3: If you took a million steps starting from the door of the classroom, how many miles would you travel? Choose a direction and determine where you would end up. How many miles would you go if you took a billion steps in the same direction? Where would you be?

Hint: 1 mile = 5,280 feet

1. In which direction do you plan to travel? __________________ Where do you estimate you will be after taking a million steps in this direction? __________________

2. One person's step is __________________ feet. When I take 10 steps, the average distance traveled is __________________ feet. Set up a proportion to find the distance (in feet) of 1,000,000 steps:

   $$\frac{10 \text{ steps}}{\_\_\_\text{feet traveled}} = \frac{1{,}000{,}000 \text{ steps}}{x \text{ feet}}$$

3. How many miles equal a million steps? __________________ miles.

4. Look at a map of your area. Where would you be after traveling a million steps north? __________________________ A million steps south? __________________________

   East? __________________________ West? __________________________

5. Now that you know how many miles you would travel after taking a million steps, how many miles would you travel after taking a billion steps? ________________________

6. The circumference of the Earth is roughly 25,000 miles. Taking a billion steps, how many times around the Earth would you go? ____________________________

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STUDENT WORKSHEET 4

Name: ____ Date: ____________________

Group #4: Imagine a crowd of a million people. How big a field do you think you would need to contain all of them? How much land area would you need to hold a billion people? If we looked at Lambeau Field in Green Bay, WI, a football stadium that can seat 81,435 people, how many football stadiums would we need to seat a million people? How many stadiums would seat a billion people?

Hint: 100 cm = 1 m

1. If an acre is about the size of a football field, my estimate of acres needed to contain a field of a million people would be ____________________ acres. The average for my group is ____________________ acres.

2. We estimate that a person needs ________ cm^2 of space. Convert this to square meters: ________ m^2

3. Set up a proportion of people to area to find how many square meters (m^2) are needed for a million people:

$$\frac{\text{1 person}}{\text{\_\_\_sq. meters (answer from \#2)}} = \frac{\text{1,000,000 people}}{\text{x sq. meters}}$$

4. If 1 acre = 4,047 square meters, about how big a field in acres would you need to contain one million people? ____________________ (acres) Can you think of an area in your community which is about this size? ____________________

5. How many square meters are needed for a billion people? ____________________ Converted into acres, it's ____________________ acres of land to contain one billion people.

6. If Lambeau Field in Green Bay, WI, a football stadium that can hold 81,435 people, we would need ____________________ stadiums to hold a million people. How many Lambeau Stadiums would we need to hold a billion people? ____________________

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STUDENT WORKSHEET 5

Name: ______________________________ Date: ____________________

Group #5: How many desks would fit in the classroom? How many classrooms would it take to hold a million desks? How many classrooms would we need to hold a billion desks?

1. Measure the volume of the desk by rounding to the nearest whole number (recall the formula for the volume of a rectangular prism is length x width x height) or use a standard desk size of 61 cm x 45 cm x 76 cm: ____________________ cm^3, or ____________________ m^3. Now find the classroom's volume by measuring its length, width, and height in meters: ____________________ m^3 (Or use a standard classroom size of 10m x 10m x 3m)

2. How many desks can you fit into this classroom (round to nearest whole number):

   ____________________________ desks

3. How many classrooms would it take to hold a million desks? Set up a proportion and solve for x.

   $$\frac{\text{(answer from \#2) desks}}{\text{1 classroom}} = \frac{\text{1,000,000 desks}}{\text{x classrooms}}$$

   so, x = ____________________ classrooms

4. How many classrooms would it take to hold a billion desks?

   ____________________________ classrooms

5. If your school has 100 classrooms in it, how many schools would you need to hold the billion desks?

   ______________________ schools